Table 4.1
Piezoelectrics under shock compression

Authors	Date	Reference	Stress, GPa	Results
Tourmaline				
Minshall	(1955)	[55M2]	7 to 10	shock arrival only
Goranson et al.	(1955)	[55G1]	1.6 to 32	thin sample technique
Bancroft et al.	(1956)	[56B1]	1.6 to 20	thin sample technique
Hearst et al.	(1964)	[64H1]	0.7 GPa	thin sample technique
Hearst et al.	(1965)	[65H2]	2.1 GPa	thin sample technique
Quartz				
Neilson et al.	(1961)	[62N2]	0.8 to 30	electrical and optical response
Graham	(1961)	[61G1]	0.8 to 4.5	linear to 2.5 GPa
Graham	(1961)	[61G2]	0.5 to 7	gun technique
Fowles	(1961)	[61F3]	4 to 23	HEL, strength loss
Graham	(1962)	[62G1]	0.5 to 5	minus-*x* anomaly
Neilson and Benedick	(1962)	[62N1]	2.5 to 30	three-zone model
Wackerle	(1962)	[62W1]	4 to 70	HEL, strength loss
Graham et al.	(1965)	[65G1]	0.2 to 4.9	guard ring, material constants
Brooks	(1965)	[65B3]	3 to 27	luminescence
Jones	(1967)	[67J2]	0.6 to 2.1	79 K, material constants
Rohde and Jones	(1968)	[68R3]	0.6 to 2.1	573 K, material constants
Graham and Halpin	(1968)	[68G4]	0.6 to 2.1	dielectric breakdown, recovery
Jones and Halpin	(1968)	[68J1]	0.5 to 3	shorted guard ring
Lysne	(1972)	[72L2]		multiple reverberations
Graham and Ingram	(1972)	[72G4]	1 to 2.5	short pulse anomaly
Graham	(1972)	[72G3]	0.2 to 4	precise material constants
Chen and McCarthy	(1973)	[73C5]	theory	singular surface, nonlinear
Thurston	(1974)	[74T1]	theory	nonlinear theory, weak coupling
Graham	(1974)	[74G2]	4.5 to 13	yielding and multiple wave response
Graham	(1974)	[74G1]	0.2 to 4	nonlinear piezoelectrics
Graham	(1975)	[75G4]	1 to 3	shorted guard ring, conduction
Graham and Yang	(1975)	[75G6]	1 to 5	time delay dielectric breakdown
Graham and Chen	(1975)	[75G5]	theory	rate coupling effect
Chen et al.	(1976)	[76C2]	theory	coupled theory
Lawrence and Davison	(1977)	[77L1]	analysis	computer solutions, fully coupled
Duvall	(1977)	[77D7]	theory	piezoelectric as Maxwellian solid
Lithium niobate				
Graham	(1973)	[73G4]	0.2 to 1.4	material constants
Graham	(1974)	[74G1]	0.2 to 1.4	nonlinear piezoelectrics
Graham	(1977)	[77G6]	0.18 to 1.7	material constants
Stanton and Graham	(1977)	[77S1]	2 to 90	above HEL
Stanton and Graham	(1979)	[79S2]	2 to 44	HEL, strength loss, phase transition

accurately measured to 2.6 GPa for lithium niobate and lithium tantalate [76G4]. Gagnepaign and Besson [75G9] have studied nonlinear piezoelectric constants under uniaxial stress.

4.2.1. Elastic dielectric theory

When a stress pulse is propagated into a piezoelectric solid, the resulting strain produces a local polarization through the direct piezoelectric effect. This polarization causes an electric field to develop in the space between the electrodes and an associated current to flow an external circuit connecting the electrodes. The fields cause secondary stresses to develop in the sample through the indirect piezoelectric effect. The magnitude and distribution of the electric fields depend on the form of the stress pulse and the character of the external circuit, while details of the secondary stress depend on the electromechanical coupling coefficients and the mechanical boundary conditions. This coupling between electrical and mechanical effects, which is a fundamental characteristic of piezoelectric materials, considerably complicates analysis of their response to mechanical loads. In the present case nonlinear effects cause further complication. In fact, no fully-coupled closed form solution for nonlinear dynamic piezoelectric response has been developed, although solutions have been obtained numerically [76C2, 77L1]. Fortunately, electromechanical coupling is often weak and advantage can be taken of this fact to obtain approximate solutions that are accurate to within a few per cent.

Constitutive relations. Piezoelectric solids are characterized by constitutive relations among the stress, $\boldsymbol{t}$, strain, $\boldsymbol{\eta}$, entropy, s, electric field, $\boldsymbol{E}$, and electric displacement, $\boldsymbol{D}$. When uncoupled solutions are sought, it is convenient to express $\boldsymbol{t}$ and $\boldsymbol{D}$ as functions of $\boldsymbol{\eta}$, $\boldsymbol{E}$, and s, while fully-coupled solutions are more easily obtained from expressions for $\boldsymbol{t}$ and $\boldsymbol{E}$ as functions of $\boldsymbol{\eta}$, $\boldsymbol{D}$, and s. The formulation of nonlinear piezoelectric constitutive relations has been considered by numerous authors (see the list cited in [77G6]) but there is no generally-accepted form or notation. With some modification in notation, we adopt the definitions of thermodynamic potentials developed by Thurston [74T1]. This leads to the constitutive relations

$$t_{rs} = \frac{\rho}{\rho_{\mathrm{R}}} F_{ri}F_{sj}(C^{E}_{ijkl}\eta_{kl} - e_{kij}E_k + \tfrac{1}{2}C^{E}_{ijklmn}\eta_{kl}\eta_{mn} + \tfrac{1}{6}C^{E}_{ijklmnpq}\eta_{kl}\eta_{mn}\eta_{pq} - \tfrac{1}{2}f_{klij}E_kE_l$$
$$- \tfrac{1}{2}e_{ijklm}E_m\eta_{kl})$$
$$D_i = e_{ijk}\eta_{jk} + \varepsilon^{\eta}_{ij}E_j + \tfrac{1}{2}e_{ijklm}\eta_{jk}\eta_{lm} + \tfrac{1}{2}f_{ijkl}E_j\eta_{kl} + \tfrac{1}{2}\varepsilon^{\eta}_{ijk}E_jE_k \qquad (4.1)$$

in the independent variables $\boldsymbol{\eta}$, $\boldsymbol{E}$, and s (the coefficient tensors are functions of s). In these equations, the tensor components C^{E}_{ijk}, C^{E}_{ijklmn}, and $C^{E}_{ijklmnpq}$ are second-, third-, and fourth-order elastic stiffness coefficients at constant field, e_{kij} and e_{kijlm} are second- and third-order piezoelectric stress constants, ε^{η}_{ij} and ε^{η}_{ijk} are second- and third-order dielectric permittivities at constant strain, and f_{ijkl} is the electrostrictive coefficient. In applying these relations, the principal stress and strain components are positive in tension. Small pyroelectric contributions to electric displacement due to isentropic heating of certain crystals are treated in reference [77G6]. The contribution $\frac{1}{2}\varepsilon^{\eta}_{ijk}E_jE_k$ to the electric displacement of X-cut quartz is extremely small and is neglected in analysis of both quartz and lithium niobate.

For many problems it is convenient to separate the piezoelectric (i.e., strain-induced) polarization $\boldsymbol{P}^{\eta}$ from electric-field-induced polarizations by defining $\boldsymbol{D} = \boldsymbol{P}^{\eta} + \boldsymbol{\varepsilon}\boldsymbol{E}$, where $\boldsymbol{\varepsilon}$ is the permittivity tensor. When the material is constituted according to equation $(4.1)_2$ with the E^2 term omitted, we have

$$P^{\eta}_i = (e_{ijk} + \tfrac{1}{2}e_{ijklm}\eta_{lm})\eta_{jk}, \qquad \varepsilon_{ij} = \varepsilon^{\eta}_{ij} + \tfrac{1}{2}f_{ijkl}\eta_{kl}. \qquad (4.2)$$

Mechanical wave-propagation problems are analyzed on the basis of quasi-static electromagnetic conditions. This is an excellent approximation since the electromagnetic wavespeed greatly exceeds the mechanical wavespeed and the particle velocity is typically only about one-tenth of the mechanical wavespeed (see the discussion by Thurston [74T1]).

Configurations of interest are those using disk-shaped samples cut from crystals in orientations that permit plane waves of uniaxial strain to propagate through their thickness when a uniform